GYANBODH
An International Multidisciplinary Peer Reviewed Journal
ISSN:3048-9792
Volume: 3
Issue: 5
September-October: 2026

Application of Fuzzy Optimization Techniques to Multi-Criteria Decision-Making Problems: A Case Study Approach

Dr. Satyapal Kumar
Assistant Professor
J.P. Shikshak Prashiksan Mahavidyalaya Samastipur

Abstract

Decisions in mathematics teacher education, such as choosing an instructional strategy or distributing limited contact hours among strategies, involve several conflicting criteria and judgments that are expressed in vague, linguistic terms. Classical crisp optimization handles such imprecision poorly. This paper presents an integrated fuzzy framework that combines (i) Fuzzy TOPSIS, a fuzzy multi-criteria decision-making (MCDM) method for ranking alternatives, and (ii) Zimmermann's max-min fuzzy linear programming (FLP) for allocating a scarce resource under flexible goals and constraints. The framework is demonstrated through a case study of a mathematics pedagogy faculty team that must select and schedule instructional strategies for a mathematics methods course in a teacher education programme. Three alternatives (lecture-demonstration, problem-based learning, and technology-integrated instruction using dynamic geometry software) are evaluated on four criteria by triangular fuzzy numbers. Fuzzy TOPSIS ranks problem-based learning first (closeness coefficient 0.3928), followed by technology-integrated instruction (0.3825) and lecture-demonstration (0.3676). The fuzzy LP model then allocates a 45-hour semester at an overall satisfaction level of 0.8308. The paper discusses the closeness of the ranking, the advantages of the fuzzy approach over a crisp model, and implications for faculty decision-making.

Keywords: fuzzy optimization, multi-criteria decision-making, Fuzzy TOPSIS, fuzzy linear programming, triangular fuzzy numbers, mathematics pedagogy, teacher education

1. Introduction

Mathematics pedagogy faculty routinely make decisions in which no single option is best on every dimension. A strategy that promotes deep conceptual understanding may demand heavy

GYANBODH
An International Multidisciplinary Peer Reviewed Journal
ISSN:3048-9792
Volume: 3
Issue: 5
September-October: 2026

preparation time; a technology-rich approach may raise engagement but also cost. In addition, the information on which such decisions rest is rarely precise. Faculty say that an approach is "highly effective" or "moderately costly", not that its effectiveness is 7.31 on a ratio scale.

Fuzzy set theory, introduced by Zadeh (1965), provides a mathematical language for this kind of imprecision. Bellman and Zadeh (1970) showed that decision-making in a fuzzy environment can be modelled by treating goals and constraints as fuzzy sets and the decision as their intersection. Since then, fuzzy extensions of classical MCDM methods (AHP, TOPSIS, VIKOR, outranking methods) and of mathematical programming have been widely applied in engineering, management and, increasingly, education.

Two families of techniques are used in this paper:

- **Fuzzy MCDM (Fuzzy TOPSIS)**, which ranks a finite set of alternatives against multiple criteria using linguistic ratings.
- **Fuzzy optimization (Fuzzy LP)**, which determines the best quantities of decision variables when the aspiration level of the objective and the availability of resources are themselves flexible.

The contribution of this paper is a compact, reproducible, two-stage framework: rank first, then allocate. It is illustrated with a fully worked case study suited to the working context of a mathematics pedagogy department.

2. Review of Related Literature

Zadeh (1965) defined fuzzy sets through membership functions that take values in the interval [0, 1]. Bellman and Zadeh (1970) formulated the fuzzy decision as the intersection of fuzzy goals and constraints and proposed maximizing the membership of this intersection. Zimmermann (1978) translated this principle into a linear programming setting: with linear membership functions, the max-min decision reduces to an ordinary linear program that maximizes the smallest membership level attained. Zimmermann (2001) gives a comprehensive treatment of the theory and its applications.

In the MCDM literature, TOPSIS was proposed by Hwang and Yoon (1981) on the idea that the chosen alternative should be closest to the ideal solution and farthest from the negative-ideal solution. Chen (2000) extended TOPSIS to group decision-making with triangular fuzzy numbers, using a vertex method to measure distance between fuzzy numbers. Fuzzy extensions

GYANBODH
An International Multidisciplinary Peer Reviewed Journal
ISSN:3048-9792
Volume: 3
Issue: 5
September-October: 2026

of the Analytic Hierarchy Process (Saaty, 1980) were developed by van Laarhoven and Pedrycz (1983) and Chang (1996).

In education, fuzzy MCDM has been proposed for evaluating teaching performance, selecting learning platforms and assessing programme quality. The India-specific policy context, in particular the emphasis of the National Education Policy 2020 on competency-based, experiential and technology-enabled learning, increases the need for transparent, evidence-informed methods that teacher educators can use when choosing among pedagogical approaches. Few studies, however, combine a ranking stage with a resource-allocation stage within a single case; this paper addresses that gap.

3. Objectives

1. To present the mathematical foundations of fuzzy optimization techniques relevant to MCDM problems.
2. To develop an integrated Fuzzy TOPSIS and Fuzzy LP framework for decisions in mathematics pedagogy.
3. To demonstrate the framework through a worked case study.
4. To compare the fuzzy allocation with its crisp counterpart and discuss implications for faculty decision-making.

4. Mathematical Preliminaries

4.1 Triangular fuzzy numbers

A triangular fuzzy number (TFN) is written $\tilde{a} = (a1, a2, a3)$ with $a1 \le a2 \le a3$. Its membership function is 0 for $x < a1$, $(x - a1)/(a2 - a1)$ for $a1 \le x \le a2$, $(a3 - x)/(a3 - a2)$ for $a2 \le x \le a3$, and 0 for $x > a3$. The value $a2$ is the most likely value; $a1$ and $a3$ are the lower and upper bounds.

The vertex distance between two TFNs $\tilde{a} = (a1, a2, a3)$ and $\tilde{b} = (b1, b2, b3)$ is

$d(\tilde{a}, \tilde{b}) = \text{sqrt}\{ (1/3) [(a1 - b1)^2 + (a2 - b2)^2 + (a3 - b3)^2] \}$.

Multiplication of a TFN by a positive TFN (used for weighting) is approximated component-wise.

GYANBODH
An International Multidisciplinary Peer Reviewed Journal
ISSN:3048-9792
Volume: 3
Issue: 5
September-October: 2026

4.2 Linguistic scale

Experts express judgments with linguistic terms that are mapped to TFNs. For ratings of alternatives on a 1–9 scale: Low (1, 3, 5), Medium (3, 5, 7), High (5, 7, 9), Very High (7, 9, 9), and Very Low (1, 1, 3). For criterion importance on a 0–1 scale: Low (0.1, 0.3, 0.5), Medium (0.3, 0.5, 0.7), High (0.5, 0.7, 0.9), Very High (0.7, 0.9, 1.0).

4.3 Fuzzy TOPSIS (Chen, 2000)

Let there be m alternatives and n criteria, with aggregated fuzzy ratings $\tilde{x}ij$ and fuzzy weights $\tilde{w}j$.

1. **Normalization.** For a benefit criterion, $\tilde{r}ij = (a_ij / c^*, b_ij / c^*, c_ij / c^*)$ with c^* = max of the upper values c_ij in that criterion. For a cost criterion, $\tilde{r}ij = (a^- / c_ij, a^- / b_ij, a^- / a_ij)$ with a^- = min of the lower values a_ij.
2. **Weighting.** $\tilde{v}ij = \tilde{r}ij \otimes \tilde{w}j$.
3. **Ideal solutions.** FPIS $A^+ = (1, 1, 1)$ and FNIS $A^- = (0, 0, 0)$ for every criterion.
4. **Distances.** $di^+ = \Sigma j\, d(\tilde{v}ij, \tilde{1})$ and $di^- = \Sigma j\, d(\tilde{v}ij, \tilde{0})$.
5. **Closeness coefficient.** $CCi = di^- / (di^+ + di^-)$. The alternative with the largest CCi is preferred.

4.4 Zimmermann's max-min fuzzy linear programming

Consider the problem: find x such that $c \cdot x \gtrsim z0$ (a fuzzy goal), $Ax \lesssim b$ (fuzzy constraints), $x \geq 0$. Let the goal have an aspiration value z1 and a lowest acceptable value z0 and let each fuzzy constraint i have a tolerance pi. With linear memberships

$\mu G(x) = (c \cdot x - z0) / (z1 - z0)$ and $\mu i(x) = (bi + pi - Ai \cdot x) / pi$,

the max-min decision is obtained by solving the crisp LP: maximize λ subject to $\mu G(x) \geq \lambda$, $\mu i(x) \geq \lambda$ for all i, $0 \leq \lambda \leq 1$, $x \geq 0$. The optimal λ^* is the degree to which the goal and all constraints are jointly satisfied.

5. Proposed Framework

Stage 1 (Ranking). Faculty experts rate each alternative on each criterion in linguistic terms; the terms are converted to TFNs, aggregated, and processed by Fuzzy TOPSIS to rank the alternatives.

GYANBODH
An International Multidisciplinary Peer Reviewed Journal
ISSN:3048-9792
Volume: 3
Issue: 5
September-October: 2026

Stage 2 (Allocation). The ranked alternatives become the decision variables of a fuzzy LP that distributes limited contact hours, subject to a fuzzy effectiveness goal and fuzzy cost limit, together with crisp pedagogical minimums.

The two stages are complementary: Stage 1 answers *which* strategies are most suitable; Stage 2 answers *how much* of each to use when resources are limited and targets are flexible.

6. Case Study

6.1 Context

A mathematics pedagogy department in a teacher education institution plans a 45-hour semester course on the teaching of secondary school geometry. The faculty team must (a) decide which instructional strategy is most suitable and (b) decide how to divide the 45 hours among strategies. Three alternatives are considered:

- **A1:** Lecture-demonstration
- **A2:** Problem-based learning (PBL)
- **A3:** Technology-integrated instruction using dynamic geometry software (for example, GeoGebra)

Four criteria are used:

- **C1:** Contribution to conceptual understanding (benefit)
- **C2:** Student-teacher engagement (benefit)
- **C3:** Implementation cost, including infrastructure (cost)
- **C4:** Faculty preparation time (cost)

6.2 Stage 1: Fuzzy TOPSIS

Criterion weights (aggregated linguistic judgments).

Criterion	Linguistic weight	TFN
C1 Conceptual understanding	Very High	(0.7, 0.9, 1.0)
C2 Engagement	High	(0.5, 0.7, 0.9)
C3 Implementation cost	Medium	(0.3, 0.5, 0.7)
C4 Preparation time	Low	(0.1, 0.3, 0.5)

Aggregated fuzzy decision matrix.

GYANBODH
An International Multidisciplinary Peer Reviewed Journal
ISSN:3048-9792
Volume: 3
Issue: 5
September-October: 2026

Alternative	C1 (benefit)	C2 (benefit)	C3 (cost)	C4 (cost)
A1 Lecture-demonstration	(3, 5, 7)	(1, 3, 5)	(1, 3, 5)	(1, 1, 3)
A2 Problem-based learning	(7, 9, 9)	(5, 7, 9)	(3, 5, 7)	(5, 7, 9)
A3 Technology-integrated	(5, 7, 9)	(7, 9, 9)	(5, 7, 9)	(3, 5, 7)

Normalized matrix. For C1 and C2, $c^* = 9$. For C3 and C4, $a^- = 1$.

Alternative	C1	C2	C3	C4
A1	(0.333, 0.556, 0.778)	(0.111, 0.333, 0.556)	(0.200, 0.333, 1.000)	(0.333, 1.000, 1.000)
A2	(0.778, 1.000, 1.000)	(0.556, 0.778, 1.000)	(0.143, 0.200, 0.333)	(0.111, 0.143, 0.200)
A3	(0.556, 0.778, 1.000)	(0.778, 1.000, 1.000)	(0.111, 0.143, 0.200)	(0.143, 0.200, 0.333)

Weighted normalized matrix.

Alternative	C1	C2	C3	C4
A1	(0.233, 0.500, 0.778)	(0.056, 0.233, 0.500)	(0.060, 0.167, 0.700)	(0.033, 0.300, 0.500)
A2	(0.544, 0.900, 1.000)	(0.278, 0.544, 0.900)	(0.043, 0.100, 0.233)	(0.011, 0.043, 0.100)
A3	(0.389, 0.700, 1.000)	(0.389, 0.700, 0.900)	(0.033, 0.071, 0.140)	(0.014, 0.060, 0.167)

Distances and closeness coefficients. With $A^+ = (1, 1, 1)$ and $A^- = (0, 0, 0)$, the vertex distances summed over the four criteria give:

Alternative	d^+	d^-	CC	Rank
A1 Lecture-demonstration	2.7958	1.6249	0.3676	3
A2 Problem-based learning	2.5933	1.6778	0.3928	1
A3 Technology-integrated	2.6318	1.6305	0.3825	2

GYANBODH
An International Multidisciplinary Peer Reviewed Journal
ISSN:3048-9792
Volume: 3
Issue: 5
September-October: 2026

Problem-based learning is the most suitable alternative (CC = 0.3928), followed by technology-integrated instruction (CC = 0.3825) and lecture-demonstration (CC = 0.3676). The absolute CC values are low because the cost criteria penalize both A2 and A3 heavily; the ranking, not the absolute value, is the quantity of interest.

6.3 Stage 2: Fuzzy LP for allocating contact hours

Let x1, x2 and x3 be the hours devoted to lecture-demonstration, PBL and technology-integrated instruction. Let the expected learning-effectiveness contribution per hour be 6, 9 and 8 respectively (expert-derived index points, consistent with the ordering from Stage 1 on the benefit criteria), and the cost per hour be 2, 5 and 6 (in thousand rupees).

Model.

- Effectiveness goal: $Z = 6x1 + 9x2 + 8x3$ should be *at least about* 345, and is unacceptable below 300. So $z1 = 345$ and $z0 = 300$.
- Cost constraint: $C = 2x1 + 5x2 + 6x3$ should be *at most about* 170, with a tolerance of 20 (cost up to 190 is tolerated at zero satisfaction).
- Crisp constraints: $x1 + x2 + x3 = 45$ (semester hours); $x1 \geq 8$ (foundational instruction); $x2 \geq 10$ (problem-solving practice); $x3 \geq 8$ (software lab sessions).

Crisp equivalent. Maximize λ subject to

$(6x1 + 9x2 + 8x3 - 300) / 45 \geq \lambda$,

$(190 - (2x1 + 5x2 + 6x3)) / 20 \geq \lambda$,

$x1 + x2 + x3 = 45$, $x1 \geq 8$, $x2 \geq 10$, $x3 \geq 8$, $0 \leq \lambda \leq 1$.

Solution. Substituting $x1 = 45 - x2 - x3$ gives $Z = 270 + 3x2 + 2x3$ and $C = 90 + 3x2 + 4x3$. The goal becomes $3x2 + 2x3 \geq 30 + 45\lambda$ and the cost constraint becomes $3x2 + 4x3 \leq 100 - 20\lambda$. Because an extra hour of PBL adds effectiveness at a lower marginal cost than an extra hour of technology-integrated instruction (3 per 3 cost units against 2 per 4 cost units), x3 stays at its minimum of 8. Setting both fuzzy constraints active:

$14 + 45\lambda = 68 - 20\lambda$, so $\lambda^* = 54/65 \approx 0.8308$.

This gives $x2 \approx 17.13$, $x3 = 8$ and $x1 \approx 19.87$.

Quantity	Fuzzy LP solution
Lecture-demonstration hours (x1)	19.9

GYANBODH
An International Multidisciplinary Peer Reviewed Journal
ISSN:3048-9792
Volume: 3
Issue: 5
September-October: 2026

Quantity	Fuzzy LP solution
Problem-based learning hours (x2)	17.1
Technology-integrated hours (x3)	8.0
Effectiveness Z	337.4
Cost C	173.4
Overall satisfaction λ*	0.8308

Rounded to practical timetable slots, the faculty team could schedule about 20, 17 and 8 hours for A1, A2 and A3.

Comparison with a crisp model. If the cost limit is treated as rigid at 170 and effectiveness is simply maximized, the crisp optimum is x1 = 21, x2 = 16, x3 = 8 with Z = 334 and C = 170. The fuzzy solution accepts a cost slightly above the nominal level (173.4) and obtains higher effectiveness (337.4). More importantly, it reports a single satisfaction degree (0.83) that makes the trade-off visible to decision-makers, rather than a hard cutoff.

7. Discussion

Consistency of the two stages. The ranking in Stage 1 places PBL first, and in Stage 2 PBL receives the highest per-hour weight in the optimal allocation among the two non-minimum options. Technology-integrated instruction is held at its pedagogical minimum because the cost constraint is binding. This agrees with the practical intuition that technology should be used where it adds distinctive value rather than as a default.

Closeness of the ranking. The gap between A2 and A3 in closeness coefficient is only 0.0103, and between A3 and A1 is 0.0149. Such small differences indicate that the ranking is sensitive to the weights and ratings. A sensitivity analysis, repeating the procedure with perturbed criterion weights, should accompany any real application. If the cost criteria receive lower weights, technology-integrated instruction would be expected to improve its relative position.

Why fuzzy methods. First, linguistic judgments are used directly, with their uncertainty preserved in the TFN spread. Second, the fuzzy LP replaces arbitrary hard limits by tolerance intervals that reflect how institutions actually negotiate budgets and targets. Third, both methods are computationally light and can be carried out in a spreadsheet, which makes them feasible for departmental use.

GYANBODH
An International Multidisciplinary Peer Reviewed Journal
ISSN:3048-9792
Volume: 3
Issue: 5
September-October: 2026

Alternative methods. Fuzzy AHP can be used where pairwise comparison of criteria is preferred to direct weighting, and VIKOR where a compromise solution is desired. The max-min operator used in the FLP is one of several aggregation operators; other operators allow compensation between goals.

8. Implications for Mathematics Pedagogy Faculty

1. **Transparent decisions.** The method documents how expert judgments lead to a decision, which supports accreditation and quality assurance reporting.
2. **Blended designs.** The allocation result supports a blended course design in which each strategy is used where it contributes most, rather than the choice of a single "best" strategy.
3. **Content for methods courses.** The framework can itself be taught to prospective mathematics teachers as an authentic application of linear programming, fuzzy sets and decision theory.
4. **Collaborative judgment.** Linguistic scales are easy for faculty and school teachers to use, which encourages participation of multiple stakeholders.

9. Limitations and Scope for Further Research

The case study uses illustrative data; the numerical results demonstrate the method and do not constitute empirical findings about the effectiveness of the strategies. The TFN weighting uses a component-wise approximation, and the LP assumes linear effectiveness and cost per hour, which ignores diminishing returns and interactions between strategies. Future work may (a) collect real expert ratings from a faculty panel, (b) test sensitivity to weights, (c) compare results with Fuzzy AHP and VIKOR, (d) use trapezoidal or intuitionistic fuzzy numbers, and (e) extend the allocation model to multiple fuzzy objectives.

10. Conclusion

This paper presented a two-stage fuzzy framework for decision problems in mathematics pedagogy: Fuzzy TOPSIS to rank alternatives under multiple criteria and Zimmermann's max-min fuzzy LP to allocate limited resources under flexible goals and constraints. In the worked case, problem-based learning ranked first (CC = 0.3928), and the optimal 45-hour allocation

GYANBODH
An International Multidisciplinary Peer Reviewed Journal
ISSN:3048-9792
Volume: 3
Issue: 5
September-October: 2026

of about 19.9, 17.1 and 8.0 hours across the three strategies attained an overall satisfaction level of 0.8308. The framework is mathematically simple, transparent and adaptable, and can support evidence-informed curriculum and instructional decisions in teacher education .

References

Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. *Management Science, 17*(4), B141–B164.

Chang, D.-Y. (1996). Applications of the extent analysis method on fuzzy AHP. *European Journal of Operational Research, 95*(3), 649–655.

Chen, C.-T. (2000). Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy Sets and Systems, 114*(1), 1–9.

Hwang, C. L., & Yoon, K. (1981). *Multiple attribute decision making: Methods and applications.* Springer.

Ministry of Education, Government of India. (2020). *National Education Policy 2020.*

Saaty, T. L. (1980). *The analytic hierarchy process.* McGraw-Hill.

van Laarhoven, P. J. M., & Pedrycz, W. (1983). A fuzzy extension of Saaty's priority theory. *Fuzzy Sets and Systems, 11*(1–3), 229–241.

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*(3), 338–353.

Zimmermann, H.-J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems, 1*(1), 45–55.

Zimmermann, H.-J. (2001). *Fuzzy set theory and its applications* (4th ed.). Kluwer Academic Publishers.